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DETECTION RATE AND SWEEP WIDTH IN VISUAL SEARCH

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Operations Evaluation Group

CENTER FOR NAVAL ANALYSES

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TO THE 1/2
POWER

W ~ 2 (2 \pi K H / V_R)^{1/2}

$\text{G}^{\circ} \text{E} \text{KH} / \text{V}^{\circ} \text{R}$, is
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INTRODUCTION

In reference 1, page 21, the search for a target located in the plane by a searcher moving in space is studied. The instantaneous probability of detection is calculated. It is:

$$\gamma(R, H) = \frac{KH}{[H^2 + R^2]^{3/2}} \quad (1)$$

In equation 1, K is a constant, H is the altitude of the searcher and R is the distance between the searcher's projection and the target (see figure 1).

The constant K is determined experimentally. According to reference 1, page 24, the sweep width for the detection function (equation 1) is:

$$W = \int_{-\infty}^{\infty} \left\{ 1 - \exp \left[\frac{-2 \left(\frac{KH}{V_R} \right)}{(H^2 + X^2)} \right] \right\} dx \quad (2)$$

In equation 2, V_R is the speed of the searcher relative to the target. The sweep width W can be measured experimentally (reference 1). Once W is known, K is found by inverting equation 2.

This inversion is not easy to do. Koopman states, "In the majority of cases, R is much larger than H " (reference 1, page 9).

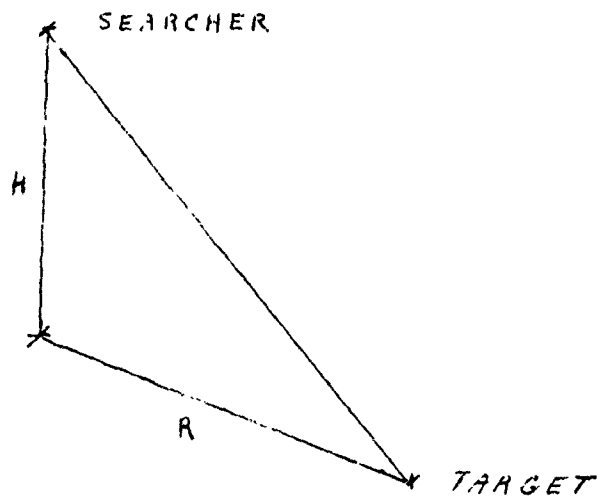


FIG. 1: GEOMETRY OF VISUAL DETECTION

He then replaces equation 1 by:

$$\gamma'(R,H) = \frac{KH}{R^3} \quad (3)$$

and equation 2 by

$$W' = \int_{-\infty}^{\infty} \left\{ 1 - \exp \left[\frac{-2 \left(\frac{KH}{V_R} \right)}{x^2} \right] \right\} dx \quad (4)$$

The integral in equation 4 is done by a change of variables. Then (reference 1, equation 29)

$$W' = 2 \sqrt{\frac{2\pi KH}{V_R}} \quad (5)$$

In order to obtain equation 4, it is assumed that $H \ll X$. But some contribution to equation 2 comes from values of X close to 0. Also, in going from equation 2 to 4, it is assumed that H is small. Aircraft searches are typically conducted at 500 feet to 5,000 feet altitudes. Is 500 feet close to zero? Visual searches from ships may have $H \approx 100$ feet.

In this paper, the integral (equation 2) is evaluated exactly- equation 5 is then approximately correct as a certain dimensionless parameter becomes large. Scaled variables are used to introduce this parameter. Although non-dimensionalization is familiar in applied mathematics, it is not used much in operations research. It should be, as the example in this paper demonstrates.

The result (equation 5) is correct when a parameter $q = KH/V_R$ is large. When W/H is large, q is large. For open ocean searches, W is typically a few miles and H is typically a fraction of a mile. Then W/H is large. There are, however, cases in which W/H is not large. In overland search or search and rescue, when the search is conducted over a forest, W may be only a few hundred yards. In this case, W/H may be small.

In the second section, the scaled variables are introduced. The third section relates the integral (equation 2) to the modified Bessel function of the first kind. The validity of Koopman's result is then verified for large q . In the fourth section, high altitude or small sweep width searches are considered. In this case, the result (equation 5) is not valid. Instead, W is proportional to H . Numerical evaluation of W is given in the last section.

INTRODUCING SCALED VARIABLES

Define non-dimensional variables $w, x, q,$

$$W = Hw$$

$$X = Hx$$

$$q = K/HV_R$$

(6)

In other words, height H is used as the length scale defining the variables. Equation 2 becomes

$$w = \int_{-\infty}^{+\infty} \left[1 - \exp \left(\frac{-2q}{1+x^2} \right) \right] dx .$$

(7)

From known W, H , we can find w . From the exact expression

$w = w(q)$ derived in the following sections, we can find q .

From q, H, V_R , we have K .

CONVERSION TO A BESSEL FUNCTION.
ASYMPTOTIC BEHAVIOR

The transformation $x = \tan \theta$, $dx = \sec^2 \theta d\theta$ converts equation 7 to

$$w(q) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[1 - \exp(-2q \cos^2 \theta) \right] \sec^2 \theta d\theta \quad (8)$$

In section 5, tables of $w(q)$ as a function of q are given. The integrand (equation 8) is even with respect to θ . Thus,

$$w(q) = 2 \int_0^{\pi/2} \left(1 - e^{-2q \cos^2 \theta} \right) \sec^2 \theta d\theta \quad (9)$$

Differentiate equation 9 with respect to q to obtain

$$\frac{dw}{dq} = 4 \int_0^{\pi/2} e^{-2q \cos^2 \theta} d\theta \quad (10)$$

Notice $w(q)$ is a strictly increasing function. Using the double angle formula

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad (11)$$

Equation 10 becomes

$$\frac{dw}{dq} = 4e^{-q} \int_0^{\pi/2} e^{-q \cos 2\theta} d\theta \quad (12)$$

Set $\alpha = 2\theta$, so that $d\theta = (1/2)d\alpha$. Equation 12 becomes

$$\frac{dw}{dq} = 2e^{-q} \int_0^{\pi} e^{-q \cos \alpha} d\alpha. \quad (13)$$

The integral in equation 13 is related to the modified Bessel function of order zero, $I_0(q)$, see reference 2. Thus,

$$\frac{dw}{dq} = 2\pi e^{-q} I_0(q). \quad (14)$$

For large q , $I_0(q)$ has the asymptotic expansion (reference 2, page 377).

$$I_0(q) \sim \frac{e^q}{\sqrt{2\pi q}} \left\{ 1 + \frac{1}{8q} + \frac{9}{2!(8q)^2} + \frac{225}{3!(8q)^3} + \dots \right\}. \quad (15)$$

Using equation 14 in equation 15 gives

$$\frac{dw}{dq} \sim \frac{2\pi}{\sqrt{2\pi q}} \left\{ 1 + \frac{1}{8q} + \frac{9}{2!(8q)^2} + \frac{225}{3!8^3 q^3} + \dots \right\}. \quad (16)$$

Integrating equation 16 gives

$$w \sim \sqrt{2\pi} \left\{ 2\sqrt{q} - \frac{1}{4} q^{-1/2} - \frac{3}{64} q^{-3/2} - \dots \right\}. \quad (17)$$

Actually, equation 17 contains a constant of integration; we show in the appendix that this constant is zero.

From equation 17, we have that $q \rightarrow +\infty$ gives large w , and it is shown in the next section that $w \rightarrow 0$ as $q \rightarrow 0$. Consequently, when $W/H = w$ becomes large, we must have large q , so equation 17 holds. In the original variables, for W/H large,

$$W \sim 2\sqrt{\frac{2\pi KH}{V_R}} - \frac{1}{4}\sqrt{\frac{2\pi V_R H^3}{K}} + \dots \quad (18)$$

The result in reference 2 is the leading term in the expansion of the exact solution. It is not necessarily valid when the altitude is small. Rather, it is valid when $\frac{W}{H} \gg 1$. It is not possible to use equation 5 to determine K when $\frac{W}{H}$ is $O(1)$ or smaller.

To determine K for a given sweep width, find the value of q that satisfies equation 13. From this value of q , K can be found. If K is known and $K/V_R H \gg 1$, then equation 5 gives the sweep width W . In the fifth section, tables of $w(q)$ are given.

HIGH ALTITUDE OR SMALL SWEEP WIDTH SEARCHES

For a high altitude search, H is large so W/H may not be large. For small q ,

$$\frac{dw}{dq} \sim 2\pi e^{-q} + \frac{\pi}{2} q^2 e^{-q} + \frac{\pi}{32} q^4 e^{-q} + \dots \quad (19)$$

and for $q \rightarrow 0$

$$w \sim 2\pi \left(1 - e^{-q}\right) \quad (20)$$

The constant of integration in equation 20 is zero because equation 8 shows $w \rightarrow 0$ as $q \rightarrow 0$. Thus,

$$W \sim 2\pi H \left(1 - e^{-K/HV_R}\right)$$

as $K/HV_R \rightarrow 0$.

NUMERICAL EVALUATION OF W

It is not advisable to use equation 5 to determine K from the sweep width. Rather, once W is known, K should be determined from equation 8. Equation 8 can be integrated numerically. In table 1, $w(q)$ is given.

A more exact evaluation of $w(q)$ is determined by power series. Define $y = 1/2\pi \cdot dw/dq$. From equation 18

$$y = e^{-q} I_0(q) . \quad (21)$$

$I_0(q)$ satisfies the equation

$$q^2 \frac{d^2 I_0}{dq^2} + q \frac{dI_0}{dq} - q^2 I_0 = 0 . \quad (22)$$

Then $y(q)$ satisfies the equation

$$q \frac{d^2 y}{dq^2} + (2q + 1) \frac{dy}{dq} + y = 0 . \quad (23)$$

Assume that

$$w(q) = 2\pi \sum_{n=1}^{\infty} a_n q^n \quad (24)$$

so that

$$y = \sum_{n=1}^{\infty} n a_n q^{n-1} \quad (25)$$

After derivatives are evaluated and substituted into equation 30, coefficients of q^n are set equal to zero. Then,

$$\begin{aligned} a_1 &= 1 \\ a_{n+1} &= - \left[\frac{2n-1}{n \cdot (n+1)} \right] a_n \end{aligned} \quad (26)$$

At $q = 10$, the exact and asymptotic results differ by 1 percent.

TABLE 1

EXACT AND ASYMPTOTIC VALUES OF $w(q)$
 $w(q) = W/H$

<u>q</u>	<u>EXACT</u>	<u>ASYMPTOTIC</u>	<u>% ERROR</u>
1	4.23	5.01	18
2	6.58	7.01	7
3	8.29	8.68	5
4	9.70	10.03	3
5	10.92	11.21	3
6	12.02	12.28	2
7	13.02	13.26	2
8	13.95	14.18	2
9	14.82	15.04	1
	15.63	15.85	1
	-	22.42	
	-	27.46	
	-	31.71	
	-	35.45	

Overflow problems.

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APPENDIX A
EVALUATION OF THE CONSTANT OF INTEGRATION

To see that the constant of integration in equation 17 is zero, substitute $y = \sqrt{q} \cos \theta$, $dy = -\sqrt{q} \sin \theta d\theta$ in equation 9 to obtain

$$w(q) = 2\sqrt{q} \int_0^{\sqrt{q}} \frac{1-e^{-2y^2}}{y^2} \frac{dy}{\sqrt{1 - \frac{y^2}{q}}} \quad (A-1)$$

Then notice for large q ,

$$\left| \int_{\frac{\sqrt{q}}{2}}^{\sqrt{q}} \frac{1-e^{-2y^2}}{y^2} \frac{dy}{\sqrt{1 - \frac{y^2}{q}}} \right| \leq \frac{1}{q/4} \int_{\frac{\sqrt{q}}{2}}^{\sqrt{q}} \frac{dy}{\sqrt{1 - \frac{y^2}{q}}} = O(1/\sqrt{q}) \quad (A-2)$$

and

$$\int_0^{\frac{\sqrt{q}}{2}} \frac{1-e^{-2y^2}}{y^2} \frac{dy}{\sqrt{1 - \frac{y^2}{q}}} = \int_0^{\infty} \frac{1-e^{-2y^2}}{y^2} dy + O(1/q), \quad (A-3)$$

so,

$$w(q) = 2 \int_0^{\infty} \frac{1-e^{-2y^2}}{y^2} dy \sqrt{q} + O(1/\sqrt{q}),$$

and no constant term appears.

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